

Используя метод разделения переменных, найти решение однородного волнового уравнения $u_{tt} = a^2 u_{xx}$, $0 < x < l$, $t > 0$ при следующих граничных и начальных условиях:

1. $u(0, t) = u(l, t) = 0$,
 $u(x, 0) = \sin \frac{\pi}{l} x + \sin \frac{3\pi}{l} x$, $u_t(x, 0) = 0$.
2. $u_x(0, t) = u(l, t) = 0$,
 $u(x, 0) = 0$, $u_t(x, 0) = 1$.
3. $u(0, t) = u_x(l, t) = 0$,
 $u(x, 0) = \sin \frac{\pi}{2l} x + \sin \frac{3\pi}{2l} x$, $u_t(x, 0) = 0$.
4. $u_x(0, t) = u_x(l, t) = 0$,
 $u(x, 0) = 1$, $u_t(x, 0) = 1$.
5. $u(0, t) = u(l, t) = 0$,
 $u(x, 0) = \sin \frac{2\pi}{l} x$, $u_t(x, 0) = 1$.
6. $u_x(0, t) = u_x(l, t) = 0$,
 $u(x, 0) = 0$, $u_t(x, 0) = 1 + \cos \frac{\pi}{l} x + \cos \frac{3\pi}{l} x$.
7. $u_x(0, t) = u(l, t) = 0$,
 $u(x, 0) = 0$, $u_t(x, 0) = \cos \frac{\pi}{2l} x + \cos \frac{5\pi}{2l} x$.
8. $u(0, t) = u_x(l, t) = 0$,
 $u(x, 0) = \sin \frac{5\pi}{2l} x$, $u_t(x, 0) = 1$.
9. $u_x(0, t) = u_x(l, t) = 0$,
 $u(x, 0) = U = const$, $u_t(x, 0) = V = const$.
10. $u(0, t) = u(l, t) = 0$,
 $u(x, 0) = 0$, $u_t(x, 0) = 1$.
11. $u_x(0, t) = u(l, t) = 0$,
 $u(x, 0) = \cos \frac{3\pi}{2l} x$, $u_t(x, 0) = 1$.
12. $u_x(0, t) = u_x(l, t) = 0$,
 $u(x, 0) = 1$, $u_t(x, 0) = 2 + \cos \frac{\pi}{l} x$.
13. $u(0, t) = u(l, t) = 0$,
 $u(x, 0) = \sin \frac{\pi}{l} x$, $u_t(x, 0) = \sin \frac{\pi}{l} x + \sin \frac{3\pi}{l} x$.
14. $u_x(0, t) = u(l, t) = 0$,
 $u(x, 0) = \cos \frac{\pi}{2l} x + \cos \frac{3\pi}{2l} x$, $u_t(x, 0) = \cos \frac{3\pi}{2l} x$.
15. $u(0, t) = u_x(l, t) = 0$,
 $u(x, 0) = \sin \frac{\pi}{2l} x$, $u_t(x, 0) = \sin \frac{\pi}{2l} x + \sin \frac{3\pi}{2l} x$.
16. $u_x(0, t) = u_x(l, t) = 0$,
 $u(x, 0) = 2 + \cos \frac{\pi}{l} x$, $u_t(x, 0) = 1 + \cos \frac{2\pi}{l} x$.
17. $u(0, t) = u(l, t) = 0$,
 $u(x, 0) = \sin \frac{2\pi}{l} x$, $u_t(x, 0) = x$.
18. $u_x(0, t) = u(l, t) = 0$,
 $u(x, 0) = 0$, $u_t(x, 0) = \cos \frac{3\pi}{2l} x + \cos \frac{5\pi}{2l} x$.
19. $u_x(0, t) = u_x(l, t) = 0$,
 $u(x, 0) = 1 + \cos \frac{2\pi}{l} x$, $u_t(x, 0) = \cos \frac{\pi}{l} x + \cos \frac{2\pi}{l} x$.
20. $u(0, t) = u(l, t) = 0$,
 $u(x, 0) = \sin \frac{2\pi}{l} x + \sin \frac{3\pi}{l} x$, $u_t(x, 0) = \sin \frac{2\pi}{l} x$.